

SELF-SIMILAR PROBLEM OF THE MOTION OF A  
PLANE LAYER OF HEATED MATERIAL WITH AN  
ARBITRARY EQUATION OF STATE

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The self-similar problem of the nonstationary motion of a plane layer of material in which energy from an external source is released for values of the flux density  $q_0$  on the boundary which are constant in time is considered. The self-similar variable is  $\mu = m/t$ , where  $m$  is the Lagrangian mass coordinate and  $t$  is the time. The characteristic values of the velocity, density, and pressure do not vary with time. For a self-similar problem the energy flux density  $q$  must also depend only on the self-similar variable. In this case  $q(\mu)$  can be an arbitrary function of its argument and can be given by a table. Examples are presented of actual physical processes in which the mass of the energy-release zone increases linearly with time. The equation of state can have an arbitrary form, including specification by a table. The gaseous state of matter for an arbitrary variable adiabatic exponent, the condensed state, and a two-phase state can be described. A solution of the self-similar problem is presented for the heating of a half-space bounded by a vacuum for a certain specific equation of state and various flux densities  $q_0$  and velocities  $M$  of the advance of the energy-release zone.

1. We consider the plane nonstationary motion of a material containing distributed energy sources in the hydrodynamic approximation. The corresponding system of equations has the form

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0; \quad \frac{\partial v}{\partial t} - \frac{\partial u}{\partial m} = 0; \\ \frac{\partial e}{\partial t} + p \frac{\partial v}{\partial t} = f, \end{aligned} \quad (1.1)$$

where  $u$  is the velocity,  $p$  is the pressure,  $v$  is the specific volume ( $v = 1/\rho$ , where  $\rho$  is the density),  $e$  is the interval energy per unit mass,  $t$  is the time,  $m$  is the Lagrangian mass coordinate, and  $f$  is the energy release ( $f > 0$ ) or loss ( $f < 0$ ) per unit mass per unit time. The system (1.1) must be supplemented by the equation of state

$$p = p(e, \rho), \quad (1.2)$$

which can have an arbitrary form, including specification by a table. In the special case of matter in the gaseous state,

$$p = e\rho(\gamma - 1), \quad (1.3)$$

where  $\gamma$  is the adiabatic exponent which, in turn, can be a function of  $e$  and  $\rho$ . The quantity  $f$  is related to the energy flux density  $q$  by the relation

$$f = -\partial q / \partial m. \quad (1.4)$$

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To determine  $f$  or  $q$  it is necessary to specify the energy-transport mechanism. We assume that the time dependence of the flux density through the mass has the special form

$$q = q(\mu) = q_0 q'(\mu), \quad \mu = \frac{m}{t}. \quad (1.5)$$

Here  $q_0$  is the flux density at the point  $m = 0$  and is constant in time. Equation (1.4) takes the form

$$f = \frac{F}{t}, \quad F = -q_0 \frac{dq'}{d\mu}.$$

Henceforth we assume that  $F \rightarrow 0$  as  $m \rightarrow \pm \infty$ , and that  $F(\mu)$  falls off fast enough so that its integral with respect to  $\mu$  is finite.

We assume that at time  $t = 0$  the material in each of the regions  $m > 0$  and  $m < 0$  is uniformly heated and has a constant density independently of whether it is at rest or moving with a constant velocity:

$$\begin{aligned} e=e_0, \rho=\rho_0, u=u_0, m > 0; \\ e=e_1, \rho=\rho_1, u=u_1, m < 0. \end{aligned}$$

If  $q \equiv 0$  everywhere, i.e., if there is no energy release in the material, the problem under consideration is an ordinary problem of the decay of an arbitrary discontinuity [1].

In the special case when the material is bounded by a vacuum only the half-space  $m > 0$  is considered; on its boundary the condition

$$p=0, m=0,$$

is satisfied. The other limiting case is the motion of a piston with a given velocity  $u(0, t) = u_p = \text{const}$ . In the general case the motion develops close to the boundary between two media, and shock waves, compression or rarefaction waves, are propagated from this boundary through the material. As  $m \rightarrow \infty$  we have the unperturbed state:  $e \rightarrow e_0, v \rightarrow v_0, u \rightarrow u_0$ , and as  $m \rightarrow -\infty$  we also have an unperturbed region:  $e \rightarrow e_1, v \rightarrow v_1, u \rightarrow u_1$ .

The equation of state can be different in the regions  $m > 0$  and  $m < 0$  if different materials are in contact at  $m = 0$  or if there is one material in different physical states in these regions. The equation of state can also change along any lines  $m = M_s t$ . Such lines can be shock waves propagating with constant velocity  $M_s$  on which the physical state of the material or its chemical composition varies significantly. In particular, the adiabatic exponent  $\gamma$  can be changed as a consequence of dissociation or ionization. When there is no energy release anywhere except in an infinitely narrow zone behind the front of a shock wave moving with the constant "velocity"  $M_s$  from the point  $m = 0$ , i.e.,  $F = \delta(\mu - \mu_s)$ , we have to deal with an ordinary detonation. It is known that the problem of a detonation wave is self-similar for an arbitrary equation of state [2].

The problems of the decay of an arbitrary discontinuity in a material with a certain special energy-release law (1.4) is also self-similar:

$$e = e(\mu), \quad p = p(\mu), \quad v = v(\mu), \quad u = u(\mu), \quad \mu = \frac{m}{t}.$$

We note that the characteristic values of the internal energy, density, and pressure are constant in time, which ensures self-similarity for an arbitrary equation of state (1.2).

After transforming to the self-similar variable  $\mu$  Eqs. (1.1) have the form

$$-\mu \frac{dv}{d\mu} + \frac{dp}{d\mu} = 0; \quad \mu \frac{dv}{d\mu} + \frac{du}{d\mu} = 0; \quad (1.6)$$

$$\mu \left( \frac{de}{d\mu} + p \frac{dv}{d\mu} \right) = F(\mu) \quad (1.7)$$

and are supplemented by Eq. (1.2).

We note that the velocity  $u$  can be eliminated from (1.6) to give

$$\mu^2 \frac{dv}{d\mu} + \frac{dp}{d\mu} = 0. \quad (1.8)$$

The problem is particularly simple to analyze if the energy release has the form

$$F = A\delta(u - \mu_v),$$

where  $\delta$  is the Dirac delta function and  $\mu_v$  is the coordinate of the "deflagration" front which does not necessarily coincide with the shock wave and must be specified on the basis of further physical considerations. In this case the parameters ahead of the energy-release zone are related to those behind it by a conservation law, and the motion outside the energy-release zone is adiabatic. This situation arises, for example, when intense fluxes of optical radiation act on condensed opaque material [3,4]. For relatively low flux densities the vapors formed are transparent to the incident radiation, while the radiation penetrates only small distances in condensed matter. In the limit it can be assumed that an infinitely narrow energy-release zone advances together with the evaporation wave, an infinitely narrow phase-transition zone. For a constant flux density and completely transparent vapors the evaporation wave moves with constant velocity, and behind the wave the Jouguet rule is satisfied (in general, this is not obligatory [5, 6]). In the examples presented below only volumetric energy release will be considered.

This problem was discussed in [7, 8] for the special case of the escape of an ideal gas with  $\gamma = \text{const}$  into a vacuum when the function  $F(\mu)$  has the special form

$$F = A\mu^{-\alpha}. \quad (1.9)$$

It was assumed that the vapor disperses into the vacuum behind a certain evaporation surface moving with a constant velocity within the material.

The following evaporation conditions were formulated in [7]: the evaporation surface is at the point  $m_v(t)$ , where the temperature in the condensed material rose from the initial value to a certain evaporation temperature, and energy equal to the heat of vaporation  $Q_v$  was liberated. An analytic solution of system (1.7), (1.8) was found in [7] for the flow of a gas. In this calculation the Jouguet rule was used without any justification. It was pointed out in [8] that this is a particular solution of the problem under consideration and that other distributions of parameters are possible, in particular, discontinuous distributions with a shock wave at a certain distance from the evaporation wave in the gaseous region. In this case the Jouguet rule is not satisfied on the evaporation surface. In [7, 8] the evaporation surface is understood to be an infinitely narrow zone whose existence for (1.9) is not justified. The introduction of such a discontinuity (conventional evaporation boundary) does not have any special advantages aside from the possibility of using only the simple equation of state (1.3) with  $\gamma = \text{const}$ , since it is natural to generalize such a problem [and, moreover, for an arbitrary function  $F(\mu)$  and not just for (1.9)] to the case of an arbitrary equation of state including the equation for a two-phase state. The latter enables us to consider the structure of the phase-transition zone. We note that at pressures of the order of the critical pressures (in the van der Waals sense) and higher, regions with different phases cannot generally be rigorously delimited.

We present an example of a physical process in which the energy-release zone can advance with constant velocity from the boundary of the material into its interior.

Suppose the material is heated by radiation which has a continuous spectrum, and that the spectral absorption coefficient  $\kappa_\varepsilon$  is the following function of the photon energy  $\varepsilon$ , the internal energy  $e$ , and the density  $\rho$ :

$$\kappa = K(e, \rho)\varepsilon^{-s}. \quad (1.10)$$

In this case we assume that the function  $K$  characterizing the change in transparency of the material with temperature and density is an arbitrary function of its arguments. In general, the variables in (1.10) cannot be separated. However, this may be possible to a sufficient accuracy in a certain range of temperatures and densities characteristic of the problem under discussion and in a certain (the most important) spectral range. In particular, this occurs for radiation which is hard enough so that the average photon energy is

considerably higher than the first ionization potentials; i.e., the energy is absorbed in rather deep atomic shells. Then the heating of the material to the temperature of the phase transition, its evaporation, and even its ionization, if not too strong, does not lead to a significant change in optical (in the broad sense of the word) properties of the material and the function  $K$  is constant. A power law dependence of  $\kappa$  on  $\epsilon$  is characteristic for the indicated range of photon energies [9, 10] with a typical value of  $s = 3$ .

If the form of the spectrum remains unchanged, for example, Planckian, and the characteristic energy of the source photons increases with time according to the power law

$$\epsilon_* = Bt^{1/s},$$

the characteristic absorption coefficient  $\kappa_*$  decreases with time and the characteristic mass of heated material increases linearly with time.

We note that the law of energy release in the mass in the region where most of the energy is liberated ( $q/q_0 > 0.1-0.2$ ) is described in the first crude approximation by a simple exponentially decreasing function. This means that in this region the absorption coefficient at various depths, averaged over the spectrum, does not differ too strongly from the reference value calculated from the spectrum for an effective optical thickness of the order of unity [11].

We consider another example. Recently great interest has been aroused by experiments on the pulsed heating of material using powerful electron accelerators [12]. If we restrict ourselves to relatively dense material, the energy release in it occurs mainly as a result of ionization losses, but the scattering of the electron beam must be taken into account also. In the propagation process the energy of the electrons decreases continuously from the initial value  $\epsilon_0$ . The results of calculations of corresponding problems [13-15] obtained by the Monte Carlo method for a fixed rather than a broad range of initial electron energies can be written in the form

$$q = q_0 q'(z), \quad z = Km\epsilon_0^{-s}. \quad (1.11)$$

The function  $q'(z)$  is commonly given in tabular form. Suppose the accelerator voltage and the energy of the incident electrons  $\epsilon_0$  increase with time according to the power law

$$\epsilon_0 = ct^n, \quad n = 1/s.$$

Then the energy release satisfies condition (1.5): the mass heated by electrons increases linearly with time. We note that when  $n \neq 1/s$  and/or  $q_0 \neq \text{const}$ , and  $\epsilon_*$  varies according to a power law, self-similar problems of the type [16, 17] can be formulated, but not for an arbitrary equation of state. When condition (1.5) is not satisfied, the self-similar problem under consideration can nevertheless be used to investigate the effect of both the equation of state and the law of energy release in the mass on the characteristics of the motion.

2. We consider the solution of the self-similar problem using an equation of state which approximately describes both the gaseous and condensed states:

$$p = p_x(\rho) + \rho[e - e_x(\rho)]\Gamma,$$

where we assume the dependence of the Grüneisen constant  $\Gamma$  on  $e$  and  $\rho$ , and the form of the "elastic" ("cold") components of pressure and internal energy are the same for various materials (materials of a single type described by a universal equation of state):

$$\Gamma = \Gamma(e_T^V, \rho^V), \quad e_T = c_0^2 e_T^V = e - e_x, \\ p_x = B p_x^V(\rho), \quad e_x = c_0^2 e_x^V(\rho), \quad \rho^V = \rho/\rho_0, \quad c_0^2 = Bv_0 = B/\rho_0.$$

Here  $B$  is the bulk modulus and  $\rho_0$  is the normal density. We assume that the variation of flux density  $q$  through the material is not altered by a change in the characteristic "velocity"  $M$  of propagation of the energy-release zone in the material:

$$q = q_0 q' \left( \frac{m}{Mt} \right) = q_0 q' \left( \frac{\mu}{M} \right).$$

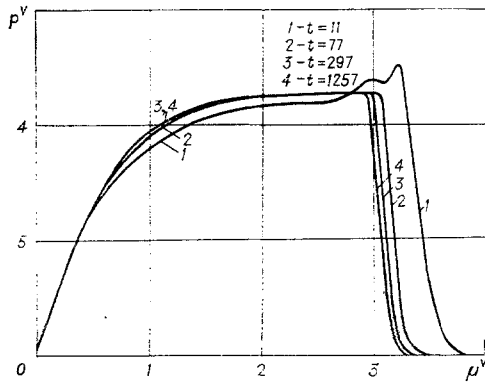


Fig. 1

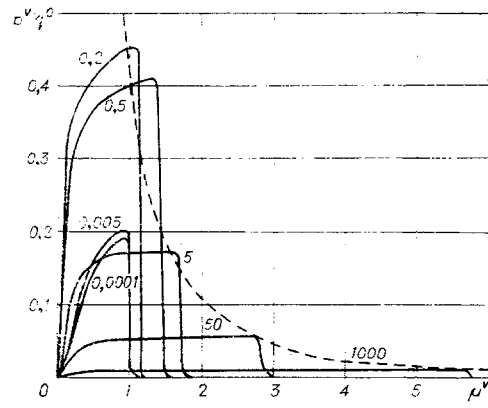


Fig. 2

We make a change of variables,

$$e = e^{\vee} c_0^2, \quad p = B p^{\vee}, \quad u = c_0 \mu^{\vee}, \\ \rho = \rho^{\vee} \rho_0, \quad q = \rho_0 c_0^2 q^{\vee}, \quad \mu = \rho_0 c_0 \mu^{\vee}.$$

It is obvious that the solution of the problem in the given form  $p_x^{\vee}(\rho^{\vee})$  and  $q'(\mu^{\vee}/M^0)$  will depend only on the parameters  $M^0$  and  $q^0$ :

$$M^0 = \frac{M}{\rho_0 c_0}, \quad q^0 = \frac{q_0}{\rho_0 c_0^3}.$$

We present the solution for an energy release described by the very simple law

$$q' = \exp\left(-\frac{\mu^{\vee}}{M^0}\right), \quad (2.1)$$

and the cold components of pressure and energy, as in [18], are given by the simple analytic expressions

$$p_x^{\vee} = \frac{1}{n-k} [\rho^n - \rho^k]; \quad e_x^{\vee} = \frac{n-k}{(n-1)(k-1)} + \frac{\rho^{n-1}}{n-1} + \frac{\rho^{k-1}}{k-1}.$$

Here for convenience in writing the superscript  $\vee$  on  $(\rho^{\vee})$  has been omitted. In the specific variants described below  $n = 3, k = 2, \Gamma = 1$ . The solutions were constructed for a wide range of values of the parameters  $M^0$  and  $q^0$ , namely,  $10^{-4} \leq q^0 \leq 10^4$ , and  $0.1 \leq M^0 \leq 10$ . According to (1.5) the heated mass is infinitesimal as  $t \rightarrow 0$ . In the examples considered above the mass actually remains finite; Eqs. (1.10) and (1.11) are not valid for too small  $\varepsilon$ . Nevertheless, for small thicknesses the energy-release zones are generally broadened by heat conduction. Therefore, using the scheme in [18] a problem which is not self-similar was calculated with the energy-release law

$$f = \exp\left(-\frac{m}{Mt + m_0}\right)$$

which goes over into (2.1) as  $t \rightarrow \infty$ .

Calculations showed that for  $t \gg m_0/M$  the solution reaches the self-similar regime. This is easy to see from Fig. 1 which shows the pressure  $p^{\vee}$  as a function of the self-similar coordinate  $\mu^{\vee}$  at different times for  $q_0 = 0.5$  and  $M^0 = 0.5$ . Curves 1-4 correspond to  $t^{\vee} = 11, 77, 297$ , and  $1257$ .

Figure 2 shows  $p^{\vee}$  as a function of  $\mu^{\vee}$  for the values of  $q^0$  noted on the curves obtained by solving the self-similar problem for  $M^0 = 1/6$ . The shock wave, which is a sound wave for small  $q^0$ , is clearly

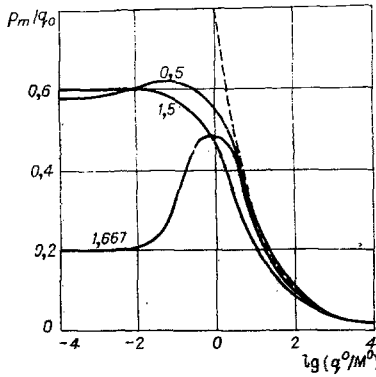


Fig. 3

visible. Its front is at the point  $\mu^V=1$ , i.e., at  $m = \rho_0 c_0 t$ . For large  $q_0$  the shock wave is strong. The open curve represents the results of scaling the value of the maximum pressure on the shock front for large  $q^0$  according to the law which holds for a pure gaseous equation of state, i.e., when the quantities  $B$  and  $C_0$  are unessential parameters. We obtain this law from qualitative arguments, although it can be found purely formally also. The characteristic density of released energy  $G$  in the energy-release zone remains constant at all times. In self-similar motion a constant fraction of the released energy is transformed into kinetic energy; i.e.,  $u \sim \sqrt{G}$ . The value of the maximum pressure can be estimated from the condition  $p_m \approx \mu u \approx \sqrt{q_0 M}$ , from which it follows that  $p_m/q_0 \sim \sqrt{M/q_0}$ .

Figure 3 shows the dependence of  $p_m/q_0$  on  $q^0/M^0$  for various  $M^0$ . The values of  $M^0$  are marked on the curves. For large  $q^0$  all the curves coincide, which is in accord with the above arguments. For small  $q^0$  the ratio  $p_m/q_0$  reaches a constant value. The presence of a maximum of  $p_m/q_0$  should be noted. This result can also be obtained by simplifying the whole problem.

3. We find the velocity of sound for an arbitrary equation of state  $p = p(e, v)$ . We have the natural relation

$$dp = p_e de + p_v dv, \quad p_e = \left. \frac{\partial p}{\partial e} \right|_v, \quad p_v = \left. \frac{\partial p}{\partial v} \right|_e. \quad (3.1)$$

For adiabatic flow

$$\left. \frac{de}{dv} \right|_s = -p. \quad (3.2)$$

It follows from (3.1) and (3.2) that

$$\rho^2 c^2 = - \left. \frac{dp}{dv} \right|_s = p p_e - p_v. \quad (3.3)$$

We transform to a system of self-similar equations. By using (3.1), from (1.8) we obtain

$$(\mu^2 + p_v) \frac{dv}{d\mu} + p_e \frac{de}{d\mu} = 0. \quad (3.4)$$

Using (3.3), from (3.4) and (1.7) we find

$$\frac{de}{d\mu} = - \frac{F}{\mu} \left[ \frac{\mu^2 - (\rho^2 c^2 - p p_e)}{\mu^2 - \rho^2 c^2} \right]. \quad (3.5)$$

We note that for the equation of state of an ideal gas (1.3) and for (3.1) when  $\Gamma = \gamma - 1 = \text{const}$  we have

$$p_e = \frac{\gamma - 1}{v}, \quad p p_e = \rho^2 \frac{p}{\rho} (\gamma - 1).$$

We now consider the case of relatively low temperatures when  $\rho c$  varies only slightly from its value  $\rho_0 c_0$  under normal conditions and the pressure  $p$  is small in comparison with  $\rho_0 c_0^2$ . Consequently, we have  $\rho^2 c^2 - p p_e \approx \rho_0 c_0^2$ . When  $M^0 \ll 1$  or  $M \ll \rho_0 c_0$  ("subsonic" propagation of the energy-release zone) most of the energy release occurs in the region  $\mu \ll \rho_0 c_0$  or  $\mu^V \ll 1$ . Therefore in the first approximation the expression in square brackets on the right-hand side of Eq. (3.5) can be set equal to unity, giving

$$- \mu \frac{de}{d\mu} = F. \quad (3.6)$$

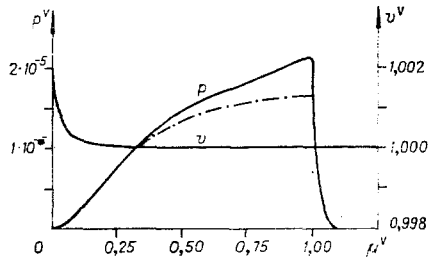


Fig. 4

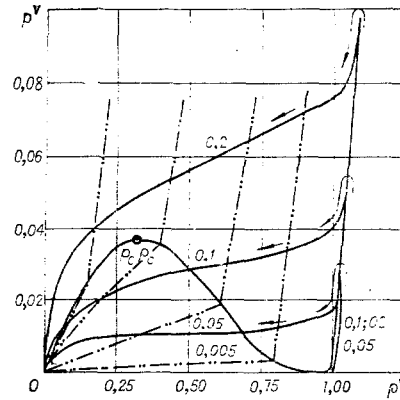


Fig. 5

Thus the motion has practically no effect on the magnitude of the internal energy;  $e$  is given by the same expression as if  $dv/d\mu = 0$  or if  $v = \text{const}$ . Substituting (3.6) into (3.4) and using the same simplifying assumptions as above, we obtain

$$p = \frac{1}{v_0} \int_0^{\mu} \frac{\mu F(\gamma-1)}{\rho_0^2 c_0^2 - \mu^2} d\mu.$$

If we consider forms of energy-release laws for which  $F$  is negligibly small in the region  $\mu \approx \rho_0 c_0$ , we obtain

$$p = \frac{(\gamma-1)qM}{\rho_0 c_0^3} H(x), \quad H(x) = \int_0^x x \left( -\frac{dq'}{dx} \right) dx. \quad (3.7)$$

Here we transform to the coordinate  $x = \mu/M$ . For  $q' = \exp(-x)$  we obtain

$$H(x) = 1 - (x+1)\exp(-x). \quad (3.8)$$

Calculations show that this expression describes the pressure profile fairly well up to the shock wave. Figure 4 shows the pressure  $p$  and the specific volume  $v$  as functions of the self-similar variable for  $M^0 = 1/6$  and  $q^0 = 10^{-4}$ . The dashed-dot curve is for Eqs. (3.7) and (3.8), and the solid curve represents the results of a numerical calculation. It is clear that the difference is appreciable only close to the shock front. We note that the density is everywhere only slightly different from normal.

It follows from (3.7) that for small  $q^0$ , i.e., in the thermoelastic region, the pressure  $p_m$  is proportional to  $q_0$ , where for  $M^0 < 1$  we obtain an increase of  $p_m$  with  $M^0$  in agreement with the data shown in Fig. 3.

When  $M^0 \gg 1$  we again use qualitative estimates. If the mass velocity of propagation of the energy-release zone is very large, the motion has practically no effect and the density does not change. The energy  $q_0 t$  released in a layer of thickness  $Mt/\rho_0$  leads to a pressure  $p_m$  of the order  $d_0 \rho_0 (\gamma-1)/M$ ; i.e.,  $p_m/q_0 \sim 1/M$  and, consequently, for large  $M^0$  the value of  $p_m/q_0$  decreases with increasing  $M$ . Accordingly, there is a certain "optimum" value of  $M$  for which the pressure is maximum. This also agrees with the data of Fig. 3.

Since the problem is self-similar, all particles of the material follow the same path on the state diagram. Figure 5 shows certain results of numerical solutions in the coordinates  $p^v, \rho^v$  for various  $q^0$  and  $M^0 = 1/6$ . For other  $M^0$  the picture is qualitatively the same. It is clear that at the beginning there is a sharp rise in pressure from the initial point  $p = 0, \rho = \rho_0$  along the curve close to the shock adiabat of the cold material, and subsequent relief for continued energy release. For a decrease in the flux density  $q^0$  to values corresponding to  $q^0 \approx 0.2$  the curves for  $p(\rho)$  pass below the critical point  $p_c, \rho_c$  through the two-phase region D. In this case they intersect this boundary twice (points a and b for the curve  $q^0 = 0.1$ ). Between such points there is a gradual quasiequilibrium volumetric evaporation and an increase in the

vapor fraction  $\eta$  from  $\eta = 0$  (condensed matter) to  $\eta = 1$  (completely vaporized matter). For comparison, the dashed-dot curves of Fig. 5 show relief adiabats [18]. It is clear that in self-similar flow with heating the behavior is very different from adiabatic, particularly outside region D, and the pressure decreases much more slowly. In order that the material be evaporated initially, i.e., in order that the curve for  $p(\rho)$  fall into region D, the material must expand. Thus, in spite of possible strong heating of material close to the shock front true evaporation can occur only in the relief region and the shock wave can move in the condensed material. For large  $q^0$  the curve for  $p(\rho)$  passes above the critical point and the concept of evaporation cannot be applied at any definite points.

In conclusion, we note that condensed matter has been treated as a fluid. The problem under consideration can be generalized by introducing both liquid and solid phases and a stress tensor instead of a single pressure.

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